## Book Reviews

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## $\mathcal{L}_1$ Adaptive Control Theory: Guaranteed Robustness with Fast Adaptation

N. Hovakimyan, and C. Cao, Society for Industrial and Applied Mathematics, Philadelphia, 2010, 337 pp., \$85 DOI: 10.2514/1.54082

This book gives a comprehensive overview of the recently developed  $\mathcal{L}_1$  adaptive control theory, with detailed proofs of the fundamental results. This new adaptive control theory offers a long awaited analytical solution to the robustness problem of adaptive control, which had remained open for the past three decades. The  $\mathcal{L}_1$  adaptive control architecture hinges on an indirect architecture of model reference adaptive control (MRAC), which enables low-pass filtering of the control signal. This, consequently, separates identification from control and enables the use of large (fast) adaptation gains without leading to high-gain control or sacrificing robustness. This fact is its key feature and main advantage, differentiating it from the MRAC direct adaptive control methods of the past, which sought to robustify the (direct control) adaptation loop with ad hoc modifications, centered around scaling the parameter adjustment law using output error magnitude in various forms (e-modification,  $\sigma$ -modification, backstepping, etc.), with no real robustness or performance guarantees. The present alternative architecture enables decoupling of the adaptation loop from robustness, which is essential in ensuring fast adaptation (corresponding to identification information processing) with guaranteed robust stability and performance. Thus, the  $\mathcal{L}_1$  theory offers a powerful tool for performance enhancement of fixed parameter baseline controllers, to which this architecture can be appended, without jeopardizing robustness. Moreover, it also offers a guaranteed performance quantification measure, including during transients.

The book has six chapters and a multipart appendix, where requisite mathematical results are summarized.

Chapter 1 offers a brief historical overview of adaptive control theory to date. It then introduces the main ideas of the  $\mathcal{L}_1$  adaptive controller. Two equivalent architectures of MRAC are analyzed, historically known as direct and indirect schemes. A stable scalar system with constant disturbance is used to introduce the main idea of the  $\mathcal{L}_1$  adaptive controller, and two key features are analyzed in detail: the closed-loop system guaranteed phase margin and the uniform bound for its control signal. Given the observation that the uniform bounds for the system state and control signal are written in terms of its impulse

response, the controller is consequently named the  $\mathcal{L}_1$  adaptive controller.

Chapter 2 presents the  $\mathcal{L}_1$  adaptive controller for systems in the presence of matched uncertainties. It starts from linear systems with constant unknown parameters, and it develops the proofs of stability and the performance bounds. The idea of decoupling adaptation from robustness is well articulated here. First, it is proved that fast adaptation ensures a uniformly bounded transient and steady-state response for both system signals (state and control) with respect to a bounded linear reference system; this naturally leads to a scaled response for the system signals in the presence of fast adaptation. Next, it is shown that the response of this linear system can be made arbitrarily close to the desired system response by tuning the low-pass filter. This step is the key to the tradeoff between performance and robustness, and it reduces to essentially tuning the structure and the bandwidth of a stable, strictly proper bandwidth-limited linear filter. Thus, the complete performance bounds of the overall nonlinear  $\mathcal{L}_1$  adaptive controller are encompassed in two terms: the first one is inversely proportional to the rate of adaptation, while the second one depends upon the bandwidth of a linear filter. This decoupling between adaptation and robustness is the key feature of the  $\mathcal{L}_1$  adaptive controller. The chapter proceeds by extending the class of applicable systems to accommodate an uncertain system input gain, time-varying parameters, and disturbances. A rigorous proof for a lower bound of the time-delay margin of the closed-loop  $\mathcal{L}_1$  adaptive system is provided in the case of open-loop linear systems with unknown constant parameters. The chapter also considers unmodeled actuator dynamics, as well as nonlinear systems in the presence of unmodeled dynamics, and uses the well known Rohrs' example to provide further insights into the  $\mathcal{L}_1$  adaptive controller. Other benchmark applications are also discussed. An overview of tuning methods for the design of this filter for the performance/robustness tradeoff is presented toward the end and, as an example, a linear-matrix-inequalitybased solution is described with certain (conservative) guarantees.

Chapter 3 extends the  $\mathcal{L}_1$  adaptive controller to accommodate unmatched uncertainties. First, nonlinear

strict-feedback systems are considered, for which the  $\mathcal{L}_1$ adaptive backstepping scheme is presented. Then, multiinput multi-output (MIMO) nonlinear systems are considered in the presence of general unmatched uncertainties. Two different adaptive laws are introduced, one of which is directly related to the sampling parameter of the CPU, as it is piecewise constant. There are certain advantages to this new type of adaptive law: it updates the parameter estimate based on the hardware (CPU) provided specification. At the sampling times, the adaptive law reduces one of the components of the identification error to zero, with the residual being proportional to the sampling interval of integration. This implies that, by increasing the sampling rate, one can reduce the influence of the residual term on the performance bounds, which are derived for the control signal and the system state and are uniform. This MIMO architecture has been applied to NASA's generic transport model, which is part of the Airborne Subscale Transport Aircraft Research (Air-STAR) system. Because of time constraints regarding publication of the book, the flight-test results, which corroborated all theoretical claims for the  $\mathcal{L}_1$  adaptive control, could not be included in the present edition; it would really be instructive to include them in a future edition of the book.

While full state accessibility was assumed in the preceding, Chap. 4 presents the output feedback solution. Two separate cases are considered for desired reference system behavior. At first, reference systems of the first order are considered, which naturally verify the strict positive real (SPR) assumption for their transfer function. Next, the methodology is extended to accommodate non-SPR reference systems. With a clear understanding of performance limitations, the proposed methodology leads to uniform performance bounds for both state and control signals of the system with respect to the same signals of the non-SPR reference systems. The most important feature of this output feedback solution is that it does not rely on system inversion; therefore, it can apply equally well to systems with nonminimum phase zeros. The two-cart benchmark example is discussed as an illustration of this extension.

Chapter 5 presents an extension to accommodate linear time-varying (LTV) reference systems. This extension is essential for practical applications. For example, in flight control, quite often the performance specifications are different at different operational conditions over the flight envelope. This leads to a time-varying reference system, the inclusion and analysis of which cannot be captured by the tools developed in prior chapters. Appropriate mathematical tools for addressing this class of systems are presented in the appendices. The chapter also presents a complete solution for nonlinear systems in the presence of unmodeled dynamics. The uniform performance bounds of the system state and the control signal are computed with respect to the corresponding signals of a LTV reference system, which meets different transient specifications at different points of the operational envelope.

Chapter 6 summarizes some of the further extensions not captured within this book, gives an overview of the applications and the flight tests that have used this theory, and states the open problems and challenges for future work. The book concludes with appendices, where basic mathematical facts are succinctly presented to support the main proofs.

The book is well written, the mathematical presentation is rigorous and elegant, and numerous examples are worked out to help the reader gain insights. It can be an excellent reference book for a graduate-level special topics course in robust adaptive control. Particularly, as its theoretical results have subsequently been consistently verified in numerous trials on a subscale commercial jet and unmanned air vehicles, it not only provides a very convincing argument on compatibility of its good theoretical results with real problem solutions, but it also suggests that the theory contained here be applied to other real systems that stand to gain no-risk performance improvement by appending this architecture to their baseline controllers. Control engineers in industry can benefit tremendously from the methods and solutions presented in this book.

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